Analog Filter Approximations: Butterworth & Chebyshev Filters

# 1. Introduction

Analog filters are widely used to process continuous-time signals. Low-pass, high-pass, band-pass, and band-stop filters can be approximated using standard filter types. Two common approximations are:  
- Butterworth filter  
- Chebyshev filter (Type I and Type II)  
  
These approximations differ in terms of their magnitude response, transition band, and ripple characteristics.

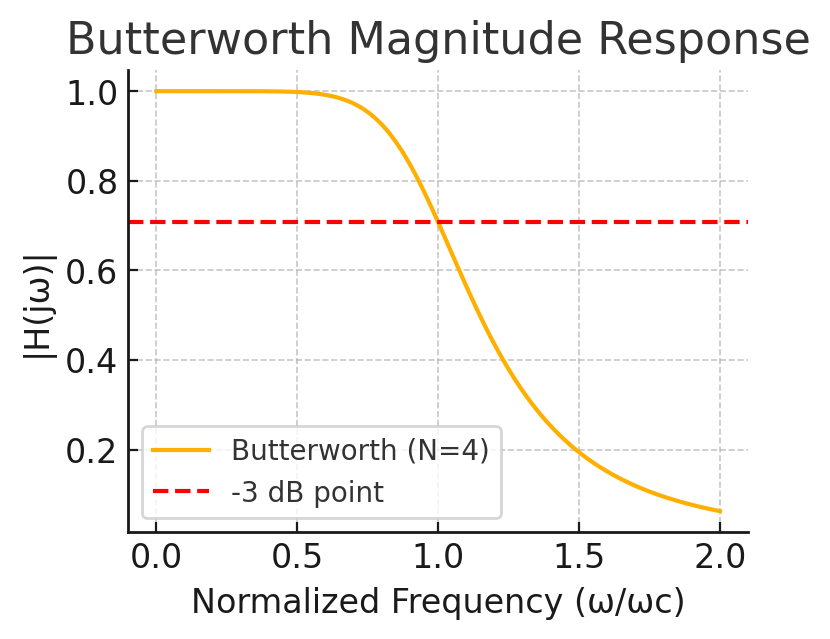
# 2. Butterworth Filter

Key Characteristics:  
- Maximally flat magnitude response in the passband  
- Monotonically decreasing in both passband and stopband  
- No ripple

Transfer Function:  
For an Nth-order Butterworth low-pass filter with cutoff frequency ωc:  
|H(jω)| = 1 / sqrt(1 + (ω/ωc)^(2N))

Magnitude Response:  
- At ω = ωc, the gain is 1/sqrt(2)  
- Roll-off: –20N dB/decade

Pole Locations:  
Poles are located on a circle in the left-half s-plane, symmetrically placed:  
s\_k = ωc \* e^(j(π/2 + (2k + 1)π / 2N)), k = 0,1,...,N-1



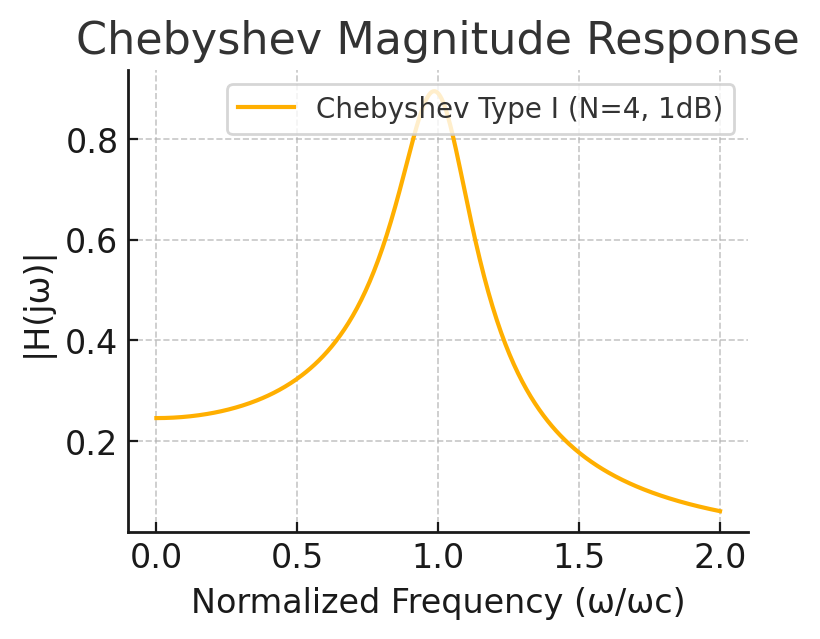
# 3. Chebyshev Filter

Two types:  
- Type I: Ripple in passband  
- Type II: Ripple in stopband (less common)

Key Characteristics (Type I):  
- Faster roll-off than Butterworth  
- Equiripple behavior in the passband  
- Non-monotonic passband

Transfer Function:  
|H(jω)| = 1 / sqrt(1 + ε^2 T\_N^2(ω/ωc))  
Where:  
- T\_N(x) is the N-th Chebyshev polynomial  
- ε is the ripple factor  
- ωc: cutoff frequency

Chebyshev Polynomial:  
T\_N(x) = cos(N \* cos⁻¹(x)), for |x| ≤ 1



# 4. Comparison Table

|  |  |  |
| --- | --- | --- |
| Feature | Butterworth | Chebyshev Type I |
| Passband Ripple | None | Yes (equiripple) |
| Stopband Ripple | None | Monotonic |
| Roll-off Rate | Slower | Faster |
| Complexity | Moderate | Higher (due to ripple) |
| Pole Distribution | Circle (s-plane) | Elliptical (s-plane) |

# 5. Conclusion

Use Butterworth filters when a smooth passband is critical.  
Use Chebyshev filters when steep roll-off is needed and ripple is acceptable.